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CTE Science Laboratory Investigation

ANGRY STUDENTS

**Introduction**

 The popular game Angry Birds is a hoot to play, but few people know that it’s also a great way to learn physics. If you think about it, you’ve got to figure out, using naught but your brain, how hard to pull back on your little birdie slingshot, as well as at what angle you want to aim it. Simple logic will tell you that the farther back you pull the slingshot, the more force you will impart to your bird. And if you aim at a really high angle, you’ll go a fair distance up, but your poor bird will land fairly close to where you started. Conversely, if you aim low, your bird might go fairly far horizontally, but won’t get very high up in the air.

 Your amazing brain can use cues (e.g. previous experience slingshotting birds) to figure out, with an astonishing degree of accuracy, what the path of the bird will be and where it will land. You do the same thing when you are throwing a football to a friend. But physics can help us calculate the trajectory a lot more accurately. We only need a few bits of information and a few formulæ and we can figure out exactly where a bird – or in our case, a water balloon – will land.

 The math in this lab isn’t tricky at all, nor are the concepts involved. There are just a few things that you must keep in mind. The most important idea is that motion in one direction is completely independent of motion in another. That is, if you shoot a bullet horizontally and drop one at the same time, both will take the same amount of time to hit the ground, but the one you fired will land further away from you. Put another way, it doesn’t matter how fast something is moving left or right, gravity will still accelerate it downwards at 9.81 m/s/s.

 The other thing that you’ll need to remember is that just as two vectors can be composed into one, you can split up one vector into two using trigonometry. In a right triangle, if you know one angle (such as the launch angle of a projectile) and length of one side (such as the initial velocity of a launch), then you can use the trigonometric relationships between that angle and the other sides of the triangle to find the lengths of the other two sides, and thus “decompose” your original vector into the horizontal and vertical components. This will prove very useful in solving for distance or time in just one direction.

**Purpose**

 The purpose of this investigation is for you to use your knowledge of physics to try and hit your teacher with a water balloon. You’ll have to take into account the mass, force and trajectory of the balloon, as well as the forces like gravity that might affect it. We will ignore air resistance in our calculations, but you can check out the assignment page on the class website for the formulae that include variables for air resistance.

**Materials**

PENCIL Water balloon

 Balloon launcher Digital balance

 Various masses Calculator

 Protractor Trundle wheel

 Target/teacher

**Procedure**

 This lab will require you to figure out a few things before you launch your balloon. You only get ONE shot to hit your teacher, and if you don’t get all your calculations right on the first go, you don’t get the points and you have to answer the analysis questions. So work carefully.

 We’ll start our lab by collecting together all the variables that you’ll need to figure out. These can be found, along with their abbreviations, in table 1.

|  |  |  |
| --- | --- | --- |
| **Variable** | **Abbreviation** | **Units** |
| Initial velocity in the vertical direction | Viy | m/s |
| Initial velocity in the horizontal direction | Vix | m/s |
| Time of flight | t | s |
| Acceleration due to gravity | g or a | m/s/s |
| Spring constant | k | N/m |
| Distance to target | d | m |
| Mass of water balloon | m | kg |

Table 1. Variables needed and their corresponding abbreviations and units.

The first thing that we need to do is figure out the spring constant of the launching apparatus. As its name implies, the spring constant is a…constant. That is, it is a characteristic of the spring that defines how springy it is. A very stiff spring, like those in the suspension of a truck, would have a very high spring constant. A spring in a watch that isn’t very strong would have a very low spring constant. It turns out that there is a formula that was developed by Robert Hooke in the 1600s that describes the spring constant in relation to a spring:

$$F=kx$$

Where F is the force applied to the spring, in newtons; k is the spring constant of the spring, in newtons per meter; and x is the distance that the spring is stretched, in meters. So we can very easily measure the spring constant of our launching device. All we need to know is the force we put on the spring and the distance that it moves.

1. Place the launching device in a vertical orientation. That is, make sure you can put a mass on the balloon launcher part and it will pull it straight down.
2. Clamp a meter stick next to the launcher so that the “0” on the meter stick is even with the balloon launcher.
3. Add a mass to the balloon launcher and measure the amount that the spring stretched.
4. Add more mass, and measure again.
5. Repeat a few more times and record your data in table 2.
6. Determine the force that you applied to the string by multiplying the mass you added by the acceleration due to gravity, 9.81 m/s/s.
7. Then calculate the spring constant using the formula given above.
8. Finally, average your spring constant and use that as your “k” for the rest of the lab.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Trial Number** | **Mass applied (kg)** | **Force\* (N)** | **Distance stretched (m)** | **Spring Constant (N/m)** |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |

Table 2. Data to determine spring constant. \*Force can easily be determined by multiplying the mass that you applied (in kg) by the acceleration due to gravity, 9.81 m/s/s.

QUESTION 1: What is your average spring constant?

Now that you have the spring constant, it’s time to meet another formula. This one is basically the spring constant formula, but backwards. Instead of telling you how much the spring will stretch if you put a certain force on it, it will tell you how fast it will shoot something out if you pull it back and let it go. The formula looks like this:

$$v\_{i}=x \sqrt{\frac{k}{m}}$$

Where vi is the initial velocity, in meters per second; x is the distance you pull back the string, in meters; k is the spring constant, in newtons per meter; and m is the mass of whatever you’re launching, in kilograms.

So now we can use this formula to figure out how fast our launcher will launch a water balloon. We can easily measure the mass of the balloon, we already calculated k, and we know how far we want to pull back the launcher. So if you put all that into the formula and solve, you can figure out the initial velocity that the balloon will have.

QUESTION 2: What is the mass of your water balloon?

QUESTION 3: How far back, in meters, will you pull the balloon launcher?

QUESTION 4: Using the spring constant that you calculated, the mass of your water balloon and the distance that you will pull back the launcher, calculate the initial velocity of the balloon as it leaves the launcher.

Now that we have the initial velocity, all we need to do is figure out how far the balloon will go. The path of the balloon will depend on what angle you use to launch it, however. So we need to break up the initial velocity into horizontal and vertical components so that we can do the math. This is pretty easy to do, however, with a bit of help from trigonometry. Consider the following vector diagram. The black arrow represents the initial velocity of the balloon that you calculated using the spring constant in the first part of this lab. If we want to find the initial horizontal velocity or the initial vertical velocity (striped arrows), all we need to do is use SOHCAHTOA.

Not tip to tail! See the correct line up in part 2…

1.

2.

vi

vi

3.

θ

viy

θ

4.

vix

The initial **horizontal** velocity is found by using the **cosine** of the angle and the initial velocity.

The initial **vertical** velocity is found by using the **sine** of the angle and the initial velocity.

Figure 1. Calculating the initial horizontal and vertical velocities for a given launch angle.

QUESTION 3: What is the angle that you will use to launch your balloon?

QUESTION 4: Calculate the initial horizontal and vertical velocities for your given angle.

 Horizontal Vertical

Let’s recap briefly, because we’re almost done but we’ve come a long way:

* You experimentally determined the spring constant of your launcher.
* You used that constant to determine the initial velocity for a given launch angle.
* You broke that initial velocity up into horizontal and vertical vectors.

Now that we have the initial velocity broken up, we just need to use a projectile motion formula to figure out how far away the balloon will land. The formula we can use is:

$$d=v\_{i}t+\frac{1}{2}at^{2}$$

Where d is the distance, in meters; vi is the initial velocity, in meters per second; a is the acceleration due to gravity, in meters per second squared; and t is the time, in seconds.

 The astute among you may notice that although this seems like a fine formula to try and use, if we substitute in what we know and try and solve for what we don’t, we end up with *two* unknown variables: distance *and* time. We can only solve for one variable whose value is unknown, so this presents us with a problem. The solution, though, is rather clever and lies in you remembering that motion in one direction (e.g. the up-and-down direction) is COMPLETELY INDEPENDENT of motion in another direction (e.g. the side-to-side direction). In other words, if we can figure out how long our balloon would take to just drop to the ground, we can use that time in our formula, because it will be exactly the same as if it had been fired out of a launcher. Motion in the side-to-side direction won’t affect how the balloon falls in any way.

 This would be fairly straightforward if we were launching the balloon horizontally, because we wouldn’t have to deal with the balloon going up first, and then coming down. In that case, we could use the formula above, substitute in zero for the initial velocity, and just use the acceleration due to gravity to determine how long the balloon would take to hit the ground. But because we are launching the balloon at an angle, we need to take one extra step.

 We’ll use another formula for projectile motion; it’s kind of the little brother of the one above, but it doesn’t take into account distance, only time. And it has another component the first one doesn’t have, viz. the final velocity of the object:

$$v\_{f}=v\_{i}+at$$

Where vf is the final velocity, in meters per second; vi is the initial velocity, also in meters per second; a is the acceleration due to gravity in meters per second squared; and t is the time, in seconds.

 We will use this formula to consider motion in only the VERTICAL direction. So we don’t care how fast the balloon is going horizontally. We already know the initial velocity in the vertical direction (you calculated it in question 4), and you know the acceleration due to gravity. But if we want to use this formula to solve for time, we also need to know the final velocity. This bit turns out to be very easy, if we remember that as an object is projected up, gravity slows it down until it stops, then starts falling. So at the very tippy-top of its arc, there is a moment where the final velocity is 0 m/s. If we substitute in 0 m/s for the final velocity, we can find how long it takes to get to the top of the arc. To figure out how long it takes to get to its final landing spot, all we need to do is double the time to include the other half of the arc. This little doubling trick works because the thing that is slowing down the balloon as it goes up is the same thing that speeds it up as it begins to fall – good ol’ Mamma Earth and her gravity.

QUESTION 5: Using the formula above, determine the time that it will take for your balloon to hit its final landing spot. Don’t forget to double the time that you get to take into account the rest of the arc!

And remember: the time that you just calculated in question five is the time that it takes to get to the landing spot for BOTH the horizontal AND vertical directions! Recall that motion in those directions is completely independent, so the time for it to go up and down is always the same, regardless of how fast it is going forward.

QUESTION 6: Using the information that you have now, calculate the horizontal distance that the balloon will travel. Use the first projectile motion formula that was given. Make sure that you use the initial velocity in the horizontal direction that you calculated in question 4!

Now you have the distance that the balloon will travel all you need to do is line up your teacher and fire away! If you hit the target (e.g. your teacher), you get a 100 on the lab and you don’t have to do any of the analysis questions. You only get one shot, so good luck!

**Analysis**

QUESTION 1: How long will it take a 10 kg ball to fall to the ground on earth if it was dropped off a 650 m tall cliff?

QUESTION 2: Bernard, who is 1.8 m tall, throws a ball horizontally off a 300 m tall cliff on the moon. He throws it with an initial horizontal velocity of 23 m/s. How long will it take to hit the ground?

QUESTION 3: In the situation in question 2, how far away from the cliff will the ball land?

QUESTION 4: If a cannonball is fired out of a cannon at 45 degrees, with an initial velocity of 65 m/s, how long will it take to land, on earth?

QUESTION 5: In the situation in question number 4, how far away from the cannon will the cannonball land?

QUESTION 6: What are two things that you would change about this lab if you could?