Differential Equations

~ an equation with a Function and a derivative

ex: $a = \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

momentum

$m \cdot v = p$

$F = ma \rightarrow F = m\frac{dv}{dt} \rightarrow F = \frac{dvm}{dt}$

$$F = \frac{dp}{dt}$$

ex: Compound Interest $\rightarrow \frac{dV}{dt} = rV$

Value = V

rate = r

$$\frac{dv}{dt} = rv$$

$$dv = rv\,dt$$

$$\frac{dv}{v} = r\,dt$$

$$\int \frac{1}{v}\,dv = \int r\,dt$$

$$\ln|v| + C = rt + D \qquad q = D - C$$

$$\ln(V) = rt + a$$

$$e^{\ln(V)} = e^{(rt+a)}$$

$$V = e^{rt+a}$$

$$V = e^{rt}e^{a}$$

e^{a} → constant → P

$$V = Pe^{rt}$$

$$V(t) = Pe^{rt}$$

ex: $\frac{dy}{dx} = 6y^2x$ $\quad y(1) = \frac{1}{25}$

$dy = 6y^2x\,dx$

$\frac{dy}{y^2} = 6x\,dx$

$\frac{1}{y^2}dy = 6x\,dx$

$\int y^{-2}dy = \int 6x\,dx$

$$\frac{y^{-1}}{-1} + C = \frac{6x^2}{2} + D$$

$$\frac{y^{-1}}{-1} + C = 3x^2 + D \qquad a = D - C$$

$$\frac{y^{-1}}{-1} = 3x^2 + a$$

$$-\frac{1}{y} = 3x^2 + a \qquad y(1) = -\frac{1}{25}$$

$$-\frac{1}{y} = 3x^2 + 9$$

$$-\frac{1}{\frac{1}{25}} = 3(1)^2 + 9$$

$$-25 = 3 + 9$$

$$-28 = 9$$

$$-\frac{1}{y} = 3x^2 + 28$$

$$y = \frac{-1}{3x^2 + 28}$$